

Constraining the cosmological constant and the DGP gravity with the double pulsar PSR J0737-3039

Lorenzo Iorio *

Istituto Nazionale di Fisica Nucleare (INFN), Sezione di Pisa. Address for correspondence: Viale Unità di Italia 68, 70125 Bari (BA), Italy

Abstract

We consider the double pulsar PSR J0737-3039A/B binary system as a laboratory to locally test the orbital effects induced by an uniform cosmological constant Λ in the framework of the known general relativistic laws of gravity, and the DGP braneworld model of gravity independently of the cosmological acceleration itself for which they were introduced. We, first, construct the ratio $R = \Delta\dot{\omega}/\Delta P$ of the discrepancies between the phenomenologically determined periastron rate $\dot{\omega}$ and orbital period P_b and their predicted values from the 1PN $\dot{\omega}_{1PN}$ approximation and the third Kepler law $P^{(0)}$. Then, we compare its value $|R| = (0.3 \pm 4) \times 10^{-11} \text{ s}^{-2}$, compatible with zero within the errors, to the ratios R_Λ and R_{DGP} of the effects induced on the apsidal rate and the orbital period by Λ and the DGP gravity; we find them neatly incompatible with R being $R_\Lambda = (3.4 \pm 0.3) \times 10^{-8} \text{ s}^{-2}$ and $R_{DGP} = (1.4 \pm 0.1) \times 10^{-7} \text{ s}^{-2}$, respectively. Such a result, which for the case of Λ is valid also for any other Hooke-like exotic force proportional to r , is in agreement with other negative local tests recently performed in the Solar System with the ratios of the non-Newtonian/Einsteinian perihelion precessions for several pairs of planets.

Key words: Experimental tests of gravitational theories; Dark energy; Modified theories of gravity; Pulsars

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1. Introduction

Since, at present, the only reason why the cosmological constant ¹ Λ is believed to be nonzero relies upon the observed acceleration of the universe (Riess et al., 1998; Perlmutter et al., 1999), i.e. just the phenomenon for which Λ was introduced (again), it is important to find independent observational tests of the existence of such an exotic component of the spacetime.

In this paper we wish to put on the test the hypothesis that $\Lambda \neq 0$, where Λ is the uniform cosmological constant of the Schwarzschild-de Sitter (Stuchlik, 1999) (or Kottler (Kottler, 1918)) spacetime, by suitably using the latest determinations of the parameters (see Table 1) of the double pulsar PSR J0737-3039A/B system (Burgay et al., 2003). The approach followed here consists in deriving analytical expressions \mathcal{O}_Λ for the effects induced by Λ on some quantities for which empirical values $\mathcal{O}_{\text{meas}}$ determined

from fitting the timing data exist. By taking into account the known classical and relativistic effects $\mathcal{O}_{\text{known}}$ affecting such quantities, the discrepancy $\Delta\mathcal{O} = \mathcal{O}_{\text{meas}} - \mathcal{O}_{\text{known}}$ is constructed and attributed to the action of Λ , which was not modelled in the pulsar data processing. Having some $\Delta\mathcal{O}$ and \mathcal{O}_Λ at hand, a suitable combination \mathcal{C} , valid just for the case $\Lambda \neq 0$, is constructed out of them in order to compare $\mathcal{C}_{\text{meas}}$ to \mathcal{C}_Λ : if the hypothesis $\Lambda \neq 0$ is correct, they must be equal within the errors. Here we will use the anomalistic period P_b and the periastron precession $\dot{\omega}$ for which purely phenomenological determinations exist in such a way that our \mathcal{C} is the ratio of $\Delta\dot{\omega}$ to ΔP_b ; as we will see, this observable is independent of Λ but, at the same time, it retains a functional dependence on the system's parameters peculiar to the Λ -induced force and of any other Hooke-like forces.

The present work complements (Iorio, 2008) in which a similar test was conducted in the Solar System by means of the latest determinations of the secular precessions of the longitudes of the perihelia of several planets. The result of (Iorio, 2008) was negative for the Schwarzschild-de Sitter spacetime with uniform Λ ; as we will see, the same conclusion will be traced out of this paper in Section 2.1.

* Corresponding author

Email address: lorenzo.iorio@libero.it (Lorenzo Iorio).

¹ See (Calder and Lahav, 2008) and references therein for an interesting historical overview.

A complementary approach to explain the cosmic acceleration without resorting to dark energy was followed by Dvali, Gabadadze and Porrati (DGP) in their braneworld modified model of gravity (Dvali et al., 2000). Among other things, it predicts effects which could be tested on a local, astronomical scale. In (Iorio, 2008) a negative test in the Solar System was reported; as we will see in Section 3, PSR J0737-3039A/B confirms such a negative outcome at a much more stringent level.

The conclusions are in Section 4.

2. The effect of Λ on the periastron and the orbital period

The Schwarzschild-de Sitter metric induces an extra-acceleration² (Rindler, 2001)

$$\mathbf{A}_\Lambda = \frac{1}{3}\Lambda c^2 \mathbf{r}, \quad (1)$$

where c is the speed of light; eq. (1), in view of the extreme smallness of the assumed nonzero value cosmological constant ($\Lambda \approx 10^{-52} \text{ m}^{-2}$), can be treated perturbatively with the standard techniques of celestial mechanics. In (Kerr et al., 2003) the secular precession of the pericentre of a test particle around a central body of mass \mathfrak{M} was found to be

$$\dot{\omega}_\Lambda = \frac{\Lambda c^2}{2n^{(0)}} \sqrt{1 - e^2}, \quad (2)$$

where

$$n^{(0)} = \sqrt{\frac{G\mathfrak{M}}{a^3}} \quad (3)$$

is the Keplerian mean motion; a and e are the semimajor axis and the eccentricity, respectively, of the test particle's orbit. Concerning a binary system, in (Jetzer and Sereno, 2006) it was shown that the equations for the relative motion are those of a test particle in a Schwarzschild-de Sitter space-time with a source mass equal to the total mass of the two-body system, i.e. $\mathfrak{M} = \mathbf{m}_A + \mathbf{m}_B$. Thus, eq. (2) is valid in our case; a is the semi-major axis of the relative orbit.

Following the approach by Jetzer and Sereno (2006), we will now compute P_Λ , i.e. the contribution of Λ to the orbital period. One of the six Keplerian orbital elements in terms of which it is possible to parameterize the orbital motion in a binary system is the mean anomaly \mathcal{M} defined as $\mathcal{M} \equiv n(t - T_0)$, where n is the mean motion and T_0 is the time of pericenter passage. The mean motion $n \equiv 2\pi/P_b$ is inversely proportional to the time elapsed between two consecutive crossings of the pericenter, i.e. the anomalistic period P_b . In Newtonian mechanics, for two point-like bodies, n reduces to the usual Keplerian expression $n^{(0)} = 2\pi/P^{(0)}$. In many binary systems, as in the double pulsar one, the period P_b is accurately determined in a phenomenological, model-independent way, so that, in principle, it accounts

for all the dynamical features of the system, not only those coming from the Newtonian point-like terms, within the measurement precision.

The Gauss equation for the variation of the mean anomaly, in the case of an entirely radial disturbing acceleration A like eq. (1), is

$$\frac{d\mathcal{M}}{dt} = n - \frac{2}{na} A \left(\frac{r}{a} \right) + \frac{(1 - e^2)}{nae} A \cos f, \quad (4)$$

where f is the true anomaly, reckoned from the pericenter. Using the eccentric anomaly E , defined as

$$\mathcal{M} = E - e \sin E, \quad (5)$$

turns out to be more convenient. The unperturbed Keplerian ellipse, on which the right-hand-side of eq. (4) must be evaluated, is

$$r = a(1 - e \cos E); \quad (6)$$

by using eq. (1) and

$$\begin{cases} \frac{d\mathcal{M}}{dE} = 1 - e \cos E, \\ \cos f = \frac{\cos E - e}{1 - e \cos E}, \end{cases} \quad (7)$$

eq. (4) becomes

$$\frac{dE}{dt} = \frac{n^{(0)}}{(1 - e \cos E)} \left\{ 1 - \frac{\Lambda c^2}{3n^{(0)2}} \left[2(1 - e \cos E)^2 - \frac{(1 - e^2)}{e} (\cos E - e) \right] \right\}. \quad (8)$$

Since $\Lambda c^2/3n^{(0)2} \approx 10^{-29}$ from eq. (8) it can be obtained

$$P_b \simeq \int_0^{2\pi} \frac{(1 - e \cos E)}{n^{(0)}} \left\{ 1 + \frac{\Lambda c^2}{3n^{(0)2}} \left[2(1 - e \cos E)^2 - \frac{(1 - e^2)}{e} (\cos E - e) \right] \right\} dE, \quad (9)$$

which integrated yields that

$$P_b = P^{(0)} + P_\Lambda \quad (10)$$

with

$$P_\Lambda = \frac{\pi \Lambda c^2 (7 + 3e^2)}{3n^{(0)3}}. \quad (11)$$

2.1. Combining the periastron and the orbital period

The general relativistic expressions of the post-Keplerian parameters r , s and $\dot{\omega}$ are

² The present test is valid for all exotic Hooke-type forces of the form Cr (Calder and Lahav, 2008), with C arbitrary nonzero constant.

$$\begin{cases} r = T_\odot m_B, \\ s = x_A \left(\frac{P_b}{2\pi} \right)^{-2/3} T_\odot^{-1/3} M^{2/3} m_B^{-1}, \\ \dot{\omega}_{\text{1PN}} = \frac{3}{(1-e^2)} \left(\frac{P_b}{2\pi} \right)^{-5/3} (T_\odot M)^{2/3}, \end{cases} \quad (12)$$

where

$$T_\odot = \frac{G\mathfrak{M}_\odot}{c^3} \quad (13)$$

and $M = m_A + m_B$ in units of solar masses.

By means of

$$a = \frac{c}{s}(x_A + x_B) \quad (14)$$

and of the equations for r and s it is possible to express $P^{(0)}$ and $\dot{\omega}_{\text{1PN}}$ in terms of P_b and of the phenomenologically determined Keplerian and post-Keplerian parameters x_A, x_B, r, s as

$$\begin{cases} P^{(0)} = 2 \left(\frac{2}{P_b} \right)^{1/2} [\pi(x_A + x_B)]^{3/2} \left(\frac{x_A}{r} \right)^{3/4} s^{-9/4}, \\ \dot{\omega}_{\text{1PN}} = \frac{3sr}{x_A(1-e^2)} \left(\frac{P_b}{2\pi} \right)^{-1}. \end{cases} \quad (15)$$

In such a way we can genuinely compare them to P_b and $\dot{\omega}$ because they do not contain quantities obtained from the third Kepler law and the general relativistic periastron precession themselves; moreover, we have expressed the sum of the masses entering both $P^{(0)}$ and $\dot{\omega}_{\text{1PN}}$ in terms of r and s , thus avoiding any possible reciprocal imprinting between the third Kepler law and the periastron rate. At this point it is possible to construct

$$R \equiv \frac{\Delta\dot{\omega}}{\Delta P}, \quad (16)$$

with

$$\begin{cases} \Delta\dot{\omega} = \dot{\omega} - \dot{\omega}_{\text{1PN}}, \\ \Delta P = P_b - P^{(0)}; \end{cases} \quad (17)$$

note that

$$R = R(P_b, x_A, x_B, e; \dot{\omega}, r, s). \quad (18)$$

By attributing $\Delta\dot{\omega}$ and ΔP to the action of Λ , not modelled into the routines used to fit the PSR J0737-3039A/B timing data, it is possible to compare R to

$$R_\Lambda \equiv \frac{\dot{\omega}_\Lambda}{P_\Lambda} = \frac{3\sqrt{1-e^2}P_b r^{3/2} s^{9/2}}{4\pi^2(7+3e^2)(x_A+x_B)^3 x_A^{3/2}} \quad (19)$$

and see if eq. (16) and eq. (19) are equal within the errors. Note that eq. (19) is independent of Λ and, by definition,

is able to test the hypothesis that $\Lambda \neq 0$. From Table 1 it turns out

$$R_\Lambda = (3.4 \pm 0.3) \times 10^{-8} \text{ s}^{-2}; \quad (20)$$

R_Λ is a well determined quantity, different from zero at about 11 sigma level. In regard to R we have

$$\begin{cases} \Delta\dot{\omega} = -0.3 \pm 2.1 \text{ deg yr}^{-1}, \\ \Delta P = 59 \pm 364 \text{ s}, \end{cases} \quad (21)$$

so that

$$|R| = (0.3 \pm 4) \times 10^{-11} \text{ s}^{-2}; \quad (22)$$

R is compatible with zero in such a way that its range does not overlap with the one of R_Λ : indeed, the upper bound on R is three orders of magnitude smaller than the lower bound on R_Λ . Thus, we must conclude that³

$$R \neq R_\Lambda. \quad (23)$$

Concerning the released uncertainties in R and R_Λ , they must be considered as upper bounds since they have been conservatively computed by linearly adding the individual biased terms due to $\delta P_b, \delta\dot{\omega}, \delta e, \delta x_A, \delta x_B, \delta r, \delta s$ in order to account for the existing correlations (Kramer et al., 2006) among them.

The results of the present study confirm those obtained in the Solar System by taking the ratio of the estimated corrections to the standard Newtonian/Einsteinian precessions of the longitude of the perihelia ϖ for different pairs of planets (Iorio, 2008). It would be very interesting to devise analogous tests involving other observables (lensing, time delay) affected by Λ as well recently computed in, e.g., (Ruggiero, 2007; Sereno, 2008).

3. The Dvali-Gabadadze-Porrati braneworld model

The approach previously outlined for Λ can be followed also for the DGP braneworld model (Dvali et al., 2000) which recently received great attention from an observational point of view (Dvali et al., 2003; Iorio, 2008).

The preliminary confrontations with data so far performed refer to the perihelia of the Solar System planets. Indeed, DGP gravity predicts an extra-precession of the pericentre of a test particle (Lue and Starkman, 2003; Iorio, 2005)

$$\dot{\omega}_{\text{DGP}} = \mp \frac{3}{8} \left(\frac{c}{r_0} \right) \left(1 - \frac{13}{32} e^2 \right), \quad (24)$$

where the signs \mp are related to the two different cosmological branches of the model and r_0 is a free-parameter set

³ In principle, also the 1PN correction to the third Kepler law (Damour and Deruelle, 1986) should be included in ΔP , but it does not change the result.

to about 5 Gpc by Type IA Supernovae data, independent of the orbit's semimajor axis. The predicted precessions of about 10^{-4} arcsec cy^{-1} were found to be compatible with the estimated corrections to the usual apsidal precessions of planets considered one at a time separately (Iorio, 2008), but marginally incompatible with the ratio of them for some pairs of inner planets (Iorio, 2007).

The effects of DGP model on the orbital period is (Iorio, 2006)

$$P_{\text{DGP}} = \mp \frac{11}{8} \pi \left(\frac{c}{r_0} \right) \frac{a^3 (1 - e^2)^2}{G\mathfrak{M}}. \quad (25)$$

From eq. (24) and eq. (25) it is possible to construct

$$R_{\text{DGP}} \equiv \frac{\dot{\omega}_{\text{DGP}}}{P_{\text{DGP}}}, \quad (26)$$

which, expressed in terms of the phenomenologically determined parameters of PSR J0737-3039A/B, becomes

$$R_{\text{DGP}} = \frac{3 \left(1 - \frac{13}{32} e^2 \right) P_b r^{3/2} s^{9/2}}{22\pi (1 - e^2) (x_A + x_B)^3 x_A^{3/2}}. \quad (27)$$

Putting the figures of Table eq. (1) into eq. (27) and computing the uncertainty as done in the case of Λ yields

$$R_{\text{DGP}} = (1.4 \pm 0.1) \times 10^{-7} \text{ s}^{-2}. \quad (28)$$

As can be noted, the lower bound of R_{DGP} is four orders of magnitude larger than the upper bound of R , so that we must conclude that, also in this case,

$$R \neq R_{\text{DGP}}. \quad (29)$$

The outcome by Iorio (2007) is, thus, confirmed at a much more stringent level.

An analysis of type Ia supernovae (SNe Ia) data disfavoring DGP model can be found in (Bento et al., 2005).

4. Conclusions

In this paper we used the most recent determinations of the orbital parameters of the double pulsar binary system PSR J0737-3039A/B to perform local tests of two complementary approaches to the issue of the observed acceleration of the universe: the uniform cosmological constant Λ in the framework of the known general relativistic laws of gravity and the multidimensional braneworld model by Dvali, Gabadadze and Porrati which, instead, resorts to a modification of the currently known laws of gravity. Since, at present, there are no observational evidences for such theoretical schemes other than just the cosmological phenomenon for which they were introduced, it is important to put them on the test independently of the cosmological acceleration itself. It is worthwhile noting that the results for Λ hold also for any other Hooke-like additional force proportional to r .

To this aim, we considered the phenomenologically determined the periastron precession $\dot{\omega}$ and the orbital period P_b of PSR J0737-3039A/B by contrasting them to the

predicted 1PN periastron rate $\dot{\omega}_{\text{1PN}}$ and the Keplerian period $P^{(0)}$. With such discrepancies we constructed the ratio $R = \Delta\omega/\Delta P$ by finding it compatible with zero: $|R| = (0.3 \pm 4) \times 10^{-11} \text{ s}^{-2}$. Then, we compared R to the predicted ratios for the effects of Λ and the DGP gravity-not modeled in the pulsar data processing-on the periastron rate and the orbital period by finding $R_\Lambda = (3.4 \pm 0.3) \times 10^{-8} \text{ s}^{-2}$ and $R_{\text{DGP}} = (1.4 \pm 0.1) \times 10^{-7} \text{ s}^{-2}$, respectively. Thus, the outcome of such a local test is neatly negative, in agreement with other local tests recently performed in the Solar System by taking the ratio of the non-Newtonian/Einsteinian rates of the perihelia for several pairs of planets.

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Table 1

Relevant Keplerian and post-Keplerian parameters of the binary system PSR J0737-3039A/B (Kramer et al., 2006). The orbital period P_b is measured with a precision of 4×10^{-6} s. The projected semimajor axis is defined as $x = (a_{bc}/c) \sin i$, where a_{bc} is the barycentric semimajor axis, c is the speed of light and i is the angle between the plane of the sky, perpendicular to the line-of-sight, and the orbital plane. The eccentricity is e . The best determined post-Keplerian parameter is, to date, the periastron rate $\dot{\omega}$ of the component A. The phenomenologically determined post-Keplerian parameter s , related to the general relativistic Shapiro time delay, is equal to $\sin i$; we have conservatively quoted the largest error in s reported in (Kramer et al., 2006). The other post-Keplerian parameter related to the Shapiro delay, which is used in the text, is r .

P_b (d)	x_A (s)	x_B (s)	e	$\dot{\omega}$ (deg yr $^{-1}$) s	r (μ s)
0.10225156248(5)	1.415032(1)	1.5161(16)	0.0877775(9)	16.89947(68)	0.99974(39) 6.21(33)